

! CHAPTER 1

DYADIC INCLUSION AND EQUIVALENCE;

! This chapter introduces the notions of inclusion and equivalence of two-place (dyadic) predicates or relationship, which correspond exactly with the unary variety. Nonetheless, they are used less frequently, and the development in this chapter is accordingly sparser.

Remark that the proofs of this chapter do not contain any appeals to propositions of the second section. i

! 1.  $\subseteq$  represents inclusion (of two-place predicates). i

$\$ \subseteq ; R \subseteq S ; \forall x \forall y (R[x,y] \Rightarrow S[x,y])$  i

! 2. i

$\vdash \forall R \forall S \forall x \forall y ( R[x,y] \ \& \ R \subseteq S \Rightarrow S[x,y] )$  i

$R, S, x, y$  ,! 1 (Prem) i

$R[x,y] \ \& \ R \subseteq S$  ,! 2 (Prem) i

$R[x,y]$  ,! 3 (&E: 2) i

$R \subseteq S$  ,! 4 (&E: 2) i

$\forall x \forall y (R[x,y] \Rightarrow S[x,y])$  ,! 5 ( $\$E$ : P1,4) i

$(R[x,y] \Rightarrow S[x,y])$  ,! 6 ( $\forall E$ : 5) i

$R[x,y] \Rightarrow S[x,y]$  ,! 7 (( $\Rightarrow$ )E: 6) i

$S[x,y]$  ,! 8 ( $\Rightarrow E$ : 3,7) i

$R[x,y] \ \& \ R \subseteq S \Rightarrow S[x,y]$  ,! 9 ( $\Rightarrow I$ : 2,8) i

$( R[x,y] \ \& \ R \subseteq S \Rightarrow S[x,y] )$  ,! 10 (( $\Rightarrow$ )I: 9) i

$\forall R \forall S \forall x \forall y ( R[x,y] \ \& \ R \subseteq S \Rightarrow S[x,y] )$  ! 11 ( $\forall I$ : 1,10) i

□

! 3. Reflexivity of Dyadic Inclusion.

i

$\vdash \forall R R \subseteq R$  i

$R$  ,! 1 (Prem) i

$x, y$  ,! 2 (Prem) i

$R[x,y]$  ,! 3 (Prem) i

$R[x,y] \Rightarrow R[x,y]$  ,! 4 ( $\Rightarrow I$ : 3,3) i

$(R[x,y] \Rightarrow R[x,y])$  ,! 5 (( $\Rightarrow$ )I: 4) i

$\forall x \forall y (R[x,y] \Rightarrow R[x,y])$	,! 6 ( $\forall I$ : 5)	i
$R \subseteq R$	,! 7 ( $\$I$ : P1,5)	i
$\forall R R \subseteq R$	! 8 ( $\forall I$ : 1,7)	i
$\square$		
<b>! 4. Transitivity of Dyadic Inclusion.</b>		i
$\vdash \forall R \forall S \forall T ( R \subseteq S \ \& \ S \subseteq T \Rightarrow R \subseteq T )$		i
$R, S, T$	,! 1 (Prem)	i
$R \subseteq S \ \& \ S \subseteq T$	,! 2 (Prem)	i
$R \subseteq S$	,! 3 ( $\&E$ : 2)	i
$\forall x \forall y (R[x,y] \Rightarrow S[x,y])$	,! 4 ( $\$E$ : P1,3)	i
$S \subseteq T$	,! 5 ( $\&E$ : 2)	i
$\forall x \forall y (S[x,y] \Rightarrow T[x,y])$	,! 6 ( $\$E$ : P1,3)	i
$x, y$	,! 7 (Prem)	i
$R[x,y]$	,! 8 (Prem)	i
$(R[x,y] \Rightarrow S[x,y])$	,! 9 ( $\forall E$ : 4)	i
$R[x,y] \Rightarrow S[x,y]$	,! 10 ( $(\Rightarrow)E$ : 9)	i
$S[x,y]$	,! 11 ( $\Rightarrow E$ : 8,10)	i
$(S[x,y] \Rightarrow T[x,y])$	,! 12 ( $\forall E$ : 6)	i
$S[x,y] \Rightarrow T[x,y]$	,! 13 ( $(\Rightarrow)E$ : 12)	i
$T[x,y]$	,! 14 ( $\Rightarrow E$ : 11,13)	i
$R[x,y] \Rightarrow T[x,y]$	,! 15 ( $\Rightarrow I$ : 8,14)	i
$(R[x,y] \Rightarrow T[x,y])$	,! 16 ( $(\Rightarrow)I$ : 15)	i
$\forall x \forall y (R[x,y] \Rightarrow T[x,y])$	,! 17 ( $\forall I$ : 7,16)	i
$R \subseteq T$	,! 18 ( $\$I$ : P1,17)	i
$R \subseteq S \ \& \ S \subseteq T \Rightarrow R \subseteq T$	,! 19 ( $\Rightarrow I$ : 2,18)	i
$( R \subseteq S \ \& \ S \subseteq T \Rightarrow R \subseteq T )$	,! 20 ( $(\Rightarrow)I$ : 19)	i
$\forall R \forall S \forall T ( R \subseteq S \ \& \ S \subseteq T \Rightarrow R \subseteq T )$	! 21 ( $\forall I$ : 1,20)	i

□

! 5.  $\equiv$  represents equivalence (of two-place predicates). i

$\$ \equiv ; R \equiv S ; \forall x \forall y (R[x,y] \leftrightarrow S[x,y])$  i

**! 6. Fundamental Proposition of Dyadic Equivalence, First Half.** i

$\vdash \forall R \forall S ( R \subseteq S \ \& \ S \subseteq R \Rightarrow R \equiv S )$  i

$R, S$  ,! 1 (Prem) i

$R \subseteq S \ \& \ S \subseteq R$  ,! 2 (Prem) i

$R \subseteq S$  ,! 3 (&E: 2) i

$\forall x \forall y (R[x,y] \Rightarrow S[x,y])$  ,! 4 ( $\$E$ : P1,3) i

$S \subseteq R$  ,! 5 (&E: 2) i

$\forall x \forall y (S[x,y] \Rightarrow R[x,y])$  ,! 6 ( $\$E$ : P1,3) i

$x, y$  ,! 7 (Prem) i

$(R[x,y] \Rightarrow S[x,y])$  ,! 8 ( $\forall E$ : 4) i

$R[x,y] \Rightarrow S[x,y]$  ,! 9 (( $\Rightarrow$ )E: 8) i

$(S[x,y] \Rightarrow R[x,y])$  ,! 10 ( $\forall E$ : 6) i

$S[x,y] \Rightarrow R[x,y]$  ,! 11 (( $\Rightarrow$ )E: 10) i

$R[x,y] \leftrightarrow S[x,y]$  ,! 12 ( $\leftrightarrow I$ : 9,11) i

$(R[x,y] \leftrightarrow S[x,y])$  ,! 13 (( $\leftrightarrow$ )I: 12) i

$\forall x \forall y (R[x,y] \leftrightarrow S[x,y])$  ,! 14 ( $\forall I$ : 13) i

$R \equiv S$  ,! 15 ( $\$I$ : P5,14) i

$R \subseteq S \ \& \ S \subseteq R \Rightarrow R \equiv S$  ,! 16 ( $\Rightarrow I$ : 2,15) i

$( R \subseteq S \ \& \ S \subseteq R \Rightarrow R \equiv S )$  ,! 17 (( $\Rightarrow$ )I: 16) i

$\forall R \forall S ( R \subseteq S \ \& \ S \subseteq R \Rightarrow R \equiv S )$  ! 18 ( $\forall I$ : 1,17) i

□

**! 7. Reflexivity of Dyadic Equivalence.** i

$\vdash \forall R R \equiv R$  i

$R$  ,! 1 (Prem) i

$R \subseteq R$  ,! 2 ( $\forall E$ : P3) i

$R \subseteq R \ \& \ R \subseteq R$	,! 3 (&I: 2,2)	i
$( R \subseteq R \ \& \ R \subseteq R \Rightarrow R \equiv R )$	,! 4 ( $\forall$ E: P6)	i
$R \subseteq R \ \& \ R \subseteq R \Rightarrow R \equiv R$	,! 5 ( $(\ )$ E: 4)	i
$R \equiv R$	,! 6 ( $\Rightarrow$ E: 3,5)	i
$\forall R R \equiv R$	! 7 ( $\forall$ I: 1,6)	i

□

**! 8. Symmetry of Dyadic Equivalence.**

$\vdash \forall R \forall S ( R \equiv S \Rightarrow S \equiv R )$		i
$R, S$	,! 1 (Prem)	i
$R \equiv S$	,! 2 (Prem)	i
$\forall x \forall y (R[x,y] \Leftrightarrow S[x,y])$	,! 3 ( $\$$ E: P5,2)	i
$x, y$	,! 4 (Prem)	i
$(R[x,y] \Leftrightarrow S[x,y])$	,! 5 ( $\forall$ E: 3)	i
$R[x,y] \Leftrightarrow S[x,y]$	,! 6 ( $(\ )$ E: 5)	i
$R[x,y] \Rightarrow S[x,y]$	,! 7 ( $\Leftrightarrow$ E: 6)	i
$S[x,y] \Rightarrow R[x,y]$	,! 8 ( $\Leftrightarrow$ E: 6)	i
$S[x,y] \Leftrightarrow R[x,y]$	,! 9 ( $\Leftrightarrow$ I: 7,8)	i
$(S[x,y] \Leftrightarrow R[x,y])$	,! 10 ( $(\ )$ I: 9)	i
$\forall x \forall y (S[x,y] \Leftrightarrow R[x,y])$	,! 11 ( $\forall$ I: 4,10)	i
$S \equiv R$	,! 12 ( $\$$ I: P5,11)	i
$R \equiv S \Rightarrow S \equiv R$	,! 13 ( $\Rightarrow$ I: 2,12)	i
$( R \equiv S \Rightarrow S \equiv R )$	,! 14 ( $(\ )$ I: 13)	i
$\forall R \forall S ( R \equiv S \Rightarrow S \equiv R )$	! 15 ( $\forall$ I: 1,14)	i

□

**! 9. Fundmantal Proposition of Dyadic Equivalence, Second Half, First Part.**

$\vdash \forall R \forall S ( R \equiv S \Rightarrow R \subseteq S )$		i
$R, S$	,! 1 (Prem)	i

$R \equiv S$	,! 2 (Prem)	i
$\forall x \forall y (R[x,y] \leftrightarrow S[x,y])$	,! 3 ( $\mathcal{SE}$ : P5,2)	i
$x, y$	,! 4 (Prem)	i
$(R[x,y] \leftrightarrow S[x,y])$	,! 5 ( $\forall E$ : 3)	i
$R[x,y] \leftrightarrow S[x,y]$	,! 6 ( $(\leftrightarrow)E$ : 5)	i
$R[x,y] \Rightarrow S[x,y]$	,! 7 ( $(\leftrightarrow)E$ : 6)	i
$(R[x,y] \Rightarrow S[x,y])$	,! 8 ( $(\Rightarrow)I$ : 7)	i
$\forall x \forall y (R[x,y] \Rightarrow S[x,y])$	,! 9 ( $\forall I$ : 4,8)	i
$R \subseteq S$	,! 10 ( $\mathcal{SI}$ : P1,9)	i
$R \equiv S \Rightarrow R \subseteq S$	,! 11 ( $(\Rightarrow)I$ : 2,10)	i
$( R \equiv S \Rightarrow R \subseteq S )$	,! 12 ( $(\Rightarrow)I$ : 11)	i
$\forall R \forall S ( R \equiv S \Rightarrow R \subseteq S )$	! 13 ( $\forall I$ : 1,12)	i

□

**! 10. Fundamental Proposition of Dyadic Equivalence, Second Half, Second Part.**

$\vdash \forall R \forall S ( R \equiv S \Rightarrow S \subseteq R )$		i
$R, S$	,! 1 (Prem)	i
$R \equiv S$	,! 2 (Prem)	i
$( R \equiv S \Rightarrow S \equiv R )$	,! 3 ( $\forall E$ : P8)	i
$R \equiv S \Rightarrow S \equiv R$	,! 4 ( $(\Rightarrow)E$ : 3)	i
$S \equiv R$	,! 5 ( $(\Rightarrow)E$ : 2,4)	i
$( S \equiv R \Rightarrow S \subseteq R )$	,! 6 ( $\forall E$ : P9)	i
$S \equiv R \Rightarrow S \subseteq R$	,! 7 ( $(\Rightarrow)E$ : 6)	i
$S \subseteq R$	,! 8 ( $(\Rightarrow)E$ : 5,7)	i
$R \equiv S \Rightarrow S \subseteq R$	,! 9 ( $(\Rightarrow)I$ : 2,8)	i
$( R \equiv S \Rightarrow S \subseteq R )$	,! 10 ( $(\Rightarrow)I$ : 9)	i
$\forall R \forall S ( R \equiv S \Rightarrow S \subseteq R )$	! 11 ( $\forall I$ : 1,10)	i

□

**! 11. Fundmental Proposition of Dyadic Equivalence, Second Half.**

$\vdash \forall R \forall S ( R \equiv S \Rightarrow R \subseteq S \ \& \ S \subseteq R )$

$R, S$	, ! 1 (Prem)	i
$R \equiv S$	, ! 2 (Prem)	i
$( R \equiv S \Rightarrow R \subseteq S )$	, ! 3 ( $\forall E$ : P9)	i
$R \equiv S \Rightarrow R \subseteq S$	, ! 4 ( $( )E$ : 3)	i
$R \subseteq S$	, ! 5 ( $\Rightarrow E$ : 2,4)	i
$( R \equiv S \Rightarrow S \subseteq R )$	, ! 6 ( $\forall E$ : P10)	i
$R \equiv S \Rightarrow S \subseteq R$	, ! 7 ( $( )E$ : 6)	i
$S \subseteq R$	, ! 8 ( $\Rightarrow E$ : 2,7)	i
$R \subseteq S \ \& \ S \subseteq R$	, ! 9 ( $\&I$ : 5,8)	i
$R \equiv S \Rightarrow R \subseteq S \ \& \ S \subseteq R$	, ! 10 ( $\Rightarrow I$ : 2,9)	i
$( R \equiv S \Rightarrow R \subseteq S \ \& \ S \subseteq R )$	, ! 11 ( $( )I$ : 10)	i

$\forall R \forall S ( R \equiv S \Rightarrow R \subseteq S \ \& \ S \subseteq R )$  ! 12 ( $\forall I$ : 1,11)

□

**! 12. Fundmental Proposition of Dyadic Equivalence.**

$\vdash \forall R \forall S ( R \subseteq S \ \& \ S \subseteq R \Leftrightarrow R \equiv S )$

$R, S$	, ! 1 (Prem)	i
$( R \subseteq S \ \& \ S \subseteq R \Rightarrow R \equiv S )$	, ! 2 ( $\forall E$ : P6)	i
$R \subseteq S \ \& \ S \subseteq R \Rightarrow R \equiv S$	, ! 3 ( $( )E$ : 2)	i
$( R \equiv S \Rightarrow R \subseteq S \ \& \ S \subseteq R )$	, ! 4 ( $\forall E$ : P11)	i
$R \equiv S \Rightarrow R \subseteq S \ \& \ S \subseteq R$	, ! 5 ( $( )E$ : 4)	i
$R \subseteq S \ \& \ S \subseteq R \Leftrightarrow R \equiv S$	, ! 6 ( $\Leftrightarrow I$ : 3,5)	i
$( R \subseteq S \ \& \ S \subseteq R \Leftrightarrow R \equiv S )$	, ! 7 ( $( )I$ : 6)	i

$\forall R \forall S ( R \subseteq S \ \& \ S \subseteq R \Leftrightarrow R \equiv S )$  ! 8 ( $\forall I$ : 1,7)

□

**! 13.**

$\vdash \forall R \forall S \forall T ( R \subseteq S \ \& \ S \equiv T \Rightarrow R \subseteq T )$		i
$R, S, T$	,! 1 (Prem)	i
$R \subseteq S \ \& \ S \equiv T$	,! 2 (Prem)	i
$R \subseteq S$	,! 3 (&E: 2)	i
$S \equiv T$	,! 4 (&E: 2)	i
$( S \equiv T \Rightarrow S \subseteq T )$	,! 5 ( $\forall$ E: P9)	i
$S \equiv T \Rightarrow S \subseteq T$	,! 6 ( $(\Rightarrow)$ E: 5)	i
$S \subseteq T$	,! 7 ( $\Rightarrow$ E: 6)	i
$R \subseteq S \ \& \ S \subseteq T$	,! 8 (&I: 3,7)	i
$( R \subseteq S \ \& \ S \subseteq T \Rightarrow R \subseteq T )$	,! 9 ( $\forall$ E: P4)	i
$R \subseteq S \ \& \ S \subseteq T \Rightarrow R \subseteq T$	,! 10 ( $(\Rightarrow)$ E: 9)	i
$R \subseteq T$	,! 11 ( $\Rightarrow$ E: 8,10)	i
$R \subseteq S \ \& \ S \equiv T \Rightarrow R \subseteq T$	,! 12 ( $\Rightarrow$ I: 2,11)	i
$( R \subseteq S \ \& \ S \equiv T \Rightarrow R \subseteq T )$	,! 13 ( $(\Rightarrow)$ I: 12)	i
$\forall R \forall S \forall T ( R \subseteq S \ \& \ S \equiv T \Rightarrow R \subseteq T )$	! 14 ( $\forall$ I: 1,13)	i

□

! 14. i

$\vdash \forall R \forall S \forall T ( R \subseteq S \ \& \ T \equiv S \Rightarrow R \subseteq T )$		i
$R, S, T$	,! 1 (Prem)	i
$R \subseteq S \ \& \ T \equiv S$	,! 2 (Prem)	i
$R \subseteq S$	,! 3 (&E: 2)	i
$T \equiv S$	,! 4 (&E: 2)	i
$( T \equiv S \Rightarrow S \equiv T )$	,! 5 ( $\forall$ E: P8)	i
$T \equiv S \Rightarrow S \equiv T$	,! 6 ( $(\Rightarrow)$ E: 5)	i
$S \equiv T$	,! 7 ( $\Rightarrow$ E: 6)	i
$R \subseteq S \ \& \ S \equiv T$	,! 8 (&I: 3,7)	i
$( R \subseteq S \ \& \ S \equiv T \Rightarrow R \subseteq T )$	,! 9 ( $\forall$ E: P13)	i
$R \subseteq S \ \& \ S \equiv T \Rightarrow R \subseteq T$	,! 10 ( $(\Rightarrow)$ E: 9)	i

$R \subseteq T$  ,! 11 ( $\Rightarrow E$ : 8,10) i  
 $R \subseteq S \ \& \ T \equiv S \Rightarrow R \subseteq T$  ,! 12 ( $\Rightarrow I$ : 2,11) i  
 $( R \subseteq S \ \& \ T \equiv S \Rightarrow R \subseteq T )$  ,! 13 ( $( ) I$ : 12) i  
 $\forall R \forall S \forall T ( R \subseteq S \ \& \ T \equiv S \Rightarrow R \subseteq T )$  ! 14 ( $\forall I$ : 1,13) i

□

**! 15. Transitivity of Dyadic Equivalence.** i

$\vdash \forall R \forall S \forall T ( R \equiv S \ \& \ S \equiv T \Rightarrow R \equiv T )$  i

$R, S, T$  ,! 1 (Prem) i  
 $R \equiv S \ \& \ S \equiv T$  ,! 2 (Prem) i  
 $R \equiv S$  ,! 3 ( $\& E$ : 2) i  
 $( R \equiv S \Rightarrow R \subseteq S \ \& \ S \subseteq R )$  ,! 4 ( $\forall E$ : P11) i  
 $R \equiv S \Rightarrow R \subseteq S \ \& \ S \subseteq R$  ,! 5 ( $( ) E$ : 4) i  
 $R \subseteq S \ \& \ S \subseteq R$  ,! 6 ( $\Rightarrow E$ : 3,5) i  
 $R \subseteq S$  ,! 7 ( $\& E$ : 6) i  
 $S \subseteq R$  ,! 8 ( $\& E$ : 6) i  
 $S \equiv T$  ,! 9 ( $\& E$ : 2) i  
 $( S \equiv T \Rightarrow S \subseteq T \ \& \ T \subseteq S )$  ,! 10 ( $\forall E$ : P11) i  
 $S \equiv T \Rightarrow S \subseteq T \ \& \ T \subseteq S$  ,! 11 ( $( ) E$ : 10) i  
 $S \subseteq T \ \& \ T \subseteq S$  ,! 12 ( $\Rightarrow E$ : 9,11) i  
 $S \subseteq T$  ,! 13 ( $\& E$ : 12) i  
 $T \subseteq S$  ,! 14 ( $\& E$ : 12) i  
 $R \subseteq S \ \& \ S \subseteq T$  ,! 15 ( $\& I$ : 7,13) i  
 $( R \subseteq S \ \& \ S \subseteq T \Rightarrow R \subseteq T )$  ,! 16 ( $\forall E$ : P4) i  
 $R \subseteq S \ \& \ S \subseteq T \Rightarrow R \subseteq T$  ,! 17 ( $( ) E$ : 16) i  
 $R \subseteq T$  ,! 18 ( $\Rightarrow E$ : 15,17) i  
 $T \subseteq S \ \& \ S \subseteq R$  ,! 19 ( $\& I$ : 8,14) i  
 $( T \subseteq S \ \& \ S \subseteq R \Rightarrow T \subseteq R )$  ,! 20 ( $\forall E$ : P4) i

$T \subseteq S \ \& \ S \subseteq R \Rightarrow T \subseteq R$  ,! 21 ((E: 20) i  
 $T \subseteq R$  ,! 22 ( $\Rightarrow$ E: 19,21) i  
 $R \subseteq T \ \& \ T \subseteq R$  ,! 23 (&I: 18,22) i  
 $( R \subseteq T \ \& \ T \subseteq R \Rightarrow R \equiv T )$  ,! 24 ( $\forall$ E: P6) i  
 $R \subseteq T \ \& \ T \subseteq R \Rightarrow R \equiv T$  ,! 25 ((E: 24) i  
 $R \equiv T$  ,! 26 ( $\Rightarrow$ E: 24,25) i  
 $R \equiv S \ \& \ S \equiv T \Rightarrow R \equiv T$  ,! 27 ( $\Rightarrow$ I: 2,26) i  
 $( R \equiv S \ \& \ S \equiv T \Rightarrow R \equiv T )$  ,! 28 ((I: 27) i  
 $\forall R \forall S \forall T ( R \equiv S \ \& \ S \equiv T \Rightarrow R \equiv T )$  ! 29 ( $\forall$ I: 1,28) i

□

! P16-P18 are applications of dyadic symmetry to transtivity. i

**! 16. Applications of Dyadic Symmetry to Transitivty, n1.** i

$\vdash \forall R \forall S \forall T ( R \equiv S \ \& \ S \equiv T \Rightarrow T \equiv R )$  i  
 $R, S, T$  ,! 1 (Prem) i  
 $R \equiv S \ \& \ S \equiv T$  ,! 2 (Prem) i  
 $( R \equiv S \ \& \ S \equiv T \Rightarrow R \equiv T )$  ,! 3 ( $\forall$ E: P15) i  
 $R \equiv S \ \& \ S \equiv T \Rightarrow R \equiv T$  ,! 4 ((E: 3) i  
 $R \equiv T$  ,! 5 ( $\Rightarrow$ E: 2,4) i  
 $( R \equiv T \Rightarrow T \equiv R )$  ,! 6 ( $\forall$ E: P8) i  
 $R \equiv T \Rightarrow T \equiv R$  ,! 7 ((E: 6) i  
 $T \equiv R$  ,! 8 ( $\Rightarrow$ E: 5,7) i  
 $R \equiv S \ \& \ S \equiv T \Rightarrow T \equiv R$  ,! 9 ( $\Rightarrow$ I: 2,8) i  
 $( R \equiv S \ \& \ S \equiv T \Rightarrow T \equiv R )$  ,! 10 ((I: 9) i  
 $\forall R \forall S \forall T ( R \equiv S \ \& \ S \equiv T \Rightarrow T \equiv R )$  ! 11 ( $\forall$ I: 1,10) i

□

**! 17. Applications of Dyadic Symmetry to Transitivty, n2.** i

$\vdash \forall R \forall S \forall T ( R \equiv S \ \& \ T \equiv S \Rightarrow R \equiv T )$  i

$R, S, T$	,! 1 (Prem)	i
$R \equiv S \ \& \ T \equiv S$	,! 2 (Prem)	i
$R \equiv S$	,! 3 (&E: 2)	i
$T \equiv S$	,! 4 (&E: 2)	i
$( T \equiv S \Rightarrow S \equiv T )$	,! 5 ( $\forall$ E: P8)	i
$T \equiv S \Rightarrow S \equiv T$	,! 6 ( $(())$ E: 5)	i
$S \equiv T$	,! 7 ( $\Rightarrow$ E: 4,6)	i
$R \equiv S \ \& \ S \equiv T$	,! 8 (&I: 3,7)	i
$( R \equiv S \ \& \ S \equiv T \Rightarrow R \equiv T )$	,! 9 ( $\forall$ E: P15)	i
$R \equiv S \ \& \ S \equiv T \Rightarrow R \equiv T$	,! 10 ( $(())$ E: 9)	i
$R \equiv T$	,! 11 ( $\Rightarrow$ E: 8,10)	i
$R \equiv S \ \& \ T \equiv S \Rightarrow R \equiv T$	,! 12 ( $\Rightarrow$ I: 2,11)	i
$( R \equiv S \ \& \ T \equiv S \Rightarrow R \equiv T )$	,! 13 ( $(())$ I: 12)	i
$\forall R \forall S \forall T ( R \equiv S \ \& \ T \equiv S \Rightarrow R \equiv T )$	! 14 ( $\forall$ I: 1,13)	i

□

**! 18. Applications of Dyadic Symmetry to Transitivity, n3.**

$\vdash \forall R \forall S \forall T ( R \equiv S \ \& \ R \equiv T \Rightarrow S \equiv T )$		i
$R, S, T$	,! 1 (Prem)	i
$R \equiv S \ \& \ R \equiv T$	,! 2 (Prem)	i
$R \equiv T$	,! 3 (&E: 2)	i
$R \equiv S$	,! 4 (&E: 2)	i
$( R \equiv S \Rightarrow S \equiv R )$	,! 5 ( $\forall$ E: P8)	i
$R \equiv S \Rightarrow S \equiv R$	,! 6 ( $(())$ E: 5)	i
$S \equiv R$	,! 7 ( $\Rightarrow$ E: 4,6)	i
$S \equiv R \ \& \ R \equiv T$	,! 8 (&I: 3,7)	i
$( S \equiv R \ \& \ R \equiv T \Rightarrow S \equiv T )$	,! 9 ( $\forall$ E: P15)	i
$S \equiv R \ \& \ R \equiv T \Rightarrow S \equiv T$	,! 10 ( $(())$ E: 9)	i

$S \equiv T$	, ! 11 ( $\Rightarrow E$ : 8,10)	i
$R \equiv S \ \& \ R \equiv T \Rightarrow S \equiv T$	, ! 12 ( $\Rightarrow I$ : 2,11)	i
$( R \equiv S \ \& \ R \equiv T \Rightarrow S \equiv T )$	, ! 13 ( $( ) I$ : 12)	i
$\forall R \forall S \forall T ( R \equiv S \ \& \ R \equiv T \Rightarrow S \equiv T )$	! 14 ( $\forall I$ : 1,13)	i

□

! 19. 3-Way Transitivity of Dyadic Equivalence. i

$\vdash \forall R \forall S \forall T \forall U ( R \equiv S \ \& \ S \equiv T \ \& \ T \equiv U \Rightarrow R \equiv U )$		i
$R, S, T, U$	, ! 1 (Prem)	i
$R \equiv S \ \& \ S \equiv T \ \& \ T \equiv U$	, ! 2 (Prem)	i
$R \equiv S \ \& \ S \equiv T$	, ! 3 ( $\& I$ : 3,4)	i
$( R \equiv S \ \& \ S \equiv T \Rightarrow R \equiv T )$	, ! 4 ( $\forall E$ : P15)	i
$R \equiv S \ \& \ S \equiv T \Rightarrow R \equiv T$	, ! 5 ( $( ) E$ : 4)	i
$R \equiv T$	, ! 6 ( $\Rightarrow E$ : 3,5)	i
$T \equiv U$	, ! 7 ( $\& E$ : 2)	i
$R \equiv T \ \& \ T \equiv U$	, ! 8 ( $\& I$ : 6,7)	i
$( R \equiv T \ \& \ T \equiv U \Rightarrow R \equiv U )$	, ! 9 ( $\forall E$ : P15)	i
$R \equiv T \ \& \ T \equiv U \Rightarrow R \equiv U$	, ! 10 ( $( ) E$ : 9)	i
$R \equiv U$	, ! 11 ( $\Rightarrow E$ : 8,10)	i
$R \equiv S \ \& \ S \equiv T \ \& \ T \equiv U \Rightarrow R \equiv U$	, ! 12 ( $\Rightarrow I$ : 2,11)	i
$( R \equiv S \ \& \ S \equiv T \ \& \ T \equiv U \Rightarrow R \equiv U )$	, ! 13 ( $( ) I$ : 12)	i
$\forall R \forall S \forall T \forall U ( R \equiv S \ \& \ S \equiv T \ \& \ T \equiv U \Rightarrow R \equiv U )$	! 14 ( $\forall I$ : 1,13)	i

□

! 20. i

$\vdash \forall R \forall S \forall x \forall y ( R[x,y] \ \& \ R \equiv S \Rightarrow S[x,y] )$		i
$R, S, x, y$	, ! 1 (Prem)	i
$R[x,y] \ \& \ R \equiv S$	, ! 2 (Prem)	i
$R[x,y]$	, ! 3 ( $\& E$ : 2)	i

$R \equiv S$	,! 4 (&E: 2)	i
$( R \equiv S \Rightarrow R \subseteq S )$	,! 5 ( $\forall$ E: P9)	i
$R \equiv S \Rightarrow R \subseteq S$	,! 6 ( $($ )E: 5)	i
$R \subseteq S$	,! 7 ( $\Rightarrow$ E: 4,6)	i
$R[x,y] \ \& \ R \subseteq S$	,! 8 (&I: 3,7)	i
$( R[x,y] \ \& \ R \subseteq S \Rightarrow S[x,y] )$	,! 9 ( $\forall$ E: P2)	i
$R[x,y] \ \& \ R \subseteq S \Rightarrow S[x,y]$	,! 10 ( $($ )E: 9)	i
$S[x,y]$	,! 11 ( $\Rightarrow$ E: 8,10)	i
$R[x,y] \ \& \ R \equiv S \Rightarrow S[x,y]$	,! 12 ( $\Rightarrow$ I: 2,11)	i
$( R[x,y] \ \& \ R \equiv S \Rightarrow S[x,y] )$	,! 13 ( $($ )I: 12)	i
$\forall R \forall S \forall x \forall y ( R[x,y] \ \& \ R \equiv S \Rightarrow S[x,y] )$	! 14 ( $\forall$ I: 1,13)	i
$\square$		
<b>! 21.</b>		i
$\vdash \forall R \forall S \forall x \forall y ( R[x,y] \ \& \ S \equiv R \Rightarrow S[x,y] )$		i
$R, S, x, y$	,! 1 (Prem)	i
$R[x,y] \ \& \ S \equiv R$	,! 2 (Prem)	i
$R[x,y]$	,! 3 (&E: 2)	i
$S \equiv R$	,! 4 (&E: 2)	i
$( S \equiv R \Rightarrow R \equiv S )$	,! 5 ( $\forall$ E: P8)	i
$S \equiv R \Rightarrow R \equiv S$	,! 6 ( $($ )E: 5)	i
$R \equiv S$	,! 7 ( $\Rightarrow$ E: 4,6)	i
$R[x,y] \ \& \ R \equiv S$	,! 8 (&I: 3,7)	i
$( R[x,y] \ \& \ R \equiv S \Rightarrow S[x,y] )$	,! 9 ( $\forall$ E: P20)	i
$R[x,y] \ \& \ R \equiv S \Rightarrow S[x,y]$	,! 10 ( $($ )E: 9)	i
$S[x,y]$	,! 11 ( $\Rightarrow$ E: 8,10)	i
$R[x,y] \ \& \ S \equiv R \Rightarrow S[x,y]$	,! 12 ( $\Rightarrow$ I: 2,11)	i
$( R[x,y] \ \& \ S \equiv R \Rightarrow S[x,y] )$	,! 13 ( $($ )I: 12)	i
$\forall R \forall S \forall x \forall y ( R[x,y] \ \& \ S \equiv R \Rightarrow S[x,y] )$	! 14 ( $\forall$ I: 1,13)	i

□